

3.

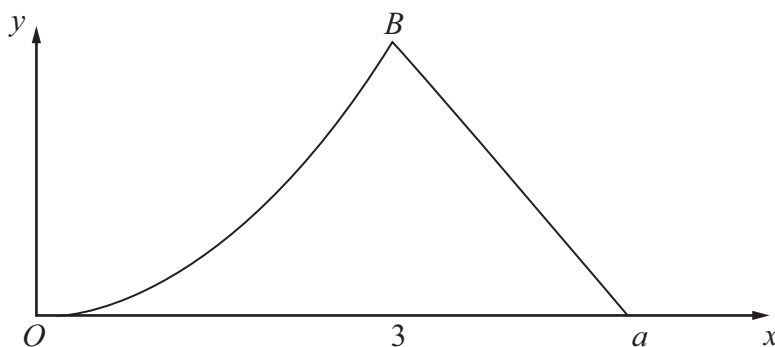


Figure 1

Figure 1 shows a sketch of the probability density function $f(x)$ of the random variable X .

For $0 \leq x \leq 3$, $f(x)$ is represented by a curve OB with equation $f(x) = kx^2$, where k is a constant.

For $3 \leq x \leq a$, where a is a constant, $f(x)$ is represented by a straight line passing through B and the point $(a, 0)$.

For all other values of x , $f(x) = 0$.

Given that the mode of $X =$ the median of X , find

- (a) the mode, (1)
- (b) the value of k , (4)
- (c) the value of a . (3)

Without calculating $E(X)$ and with reference to the skewness of the distribution

- (d) state, giving your reason, whether $E(X) < 3$, $E(X) = 3$ or $E(X) > 3$. (2)



7. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{32}(x-1)(5-x) & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f(x)$ showing clearly the points where it meets the x -axis. (2)

(b) Write down the value of the mean, μ , of X . (1)

(c) Show that $E(X^2) = 9.8$ (4)

(d) Find the standard deviation, σ , of X . (2)

The cumulative distribution function of X is given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{32}(a - 15x + 9x^2 - x^3) & 1 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

where a is a constant.

(e) Find the value of a . (2)

(f) Show that the lower quartile of X , q_1 , lies between 2.29 and 2.31 (3)

(g) Hence find the upper quartile of X , giving your answer to 1 decimal place. (1)

(h) Find, to 2 decimal places, the value of k so that

$$P(\mu - k\sigma < X < \mu + k\sigma) = 0.5 \quad (2)$$



